

HOW FAST DO SMALL x STRUCTURE FUNCTIONS RISE ? A COMPARATIVE ANALYSIS

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Abstract We parametrize the small x , singlet component of the proton structure function F_2 by powers and logarithms of $\frac{1}{x}$ for discrete values of Q^2 between 0.2 and 2000 GeV², and compare these parametrizations by applying the criterion of minimal χ^2 . The obtained values of the fitted parameters may be used to study the evolution of F_2 in Q^2 and/or in discriminating between dynamical models. A slowing-down in the increase of F_2 towards highest available values of Q^2 is revealed. The effect is quantified in terms of the derivative $\frac{d \ln F_2(x, Q^2)}{d \ln(1/x)}$.

1 Introduction

It is customary (for a recent review see, e.g. [1]) to parametrize the small x ($x < 0.1$) behaviour of the proton structure function (SF) by a power-like growth

$$F_2(x, Q^2) \sim a (Q^2) x^{-\lambda(Q^2)}, \quad (1)$$

with the Q^2 dependent "effective" power $\lambda(Q^2)$ generally interpreted as the Pomeron intercept -1 , rising from about 0.1 to about 0.4 between the smallest and largest values of Q^2 measured at HERA.

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In recent papers [2, 3] a "hard" Pomeron term, with $\epsilon_0 = 0.418$, besides the "soft" one, with $\epsilon_1 = 0.0808$ and a subleading "Reggeon" with $\epsilon_2 = -0.4525$ (all Q^2 independent) was introduced in the SF

$$F_2(x, Q^2) \sim \sum_{i=0}^2 a_i(Q^2) x^{-\epsilon_i} . \quad (2)$$

In our opinion, there is only one Pomeron in the nature [4]: nevertheless, for the sake of completeness, we have included in our analysis the above parametrization as well.

Notice that with an increasing number of contributions to F_2 the values of some of the powers (related to the intercepts of relevant trajectories) must be fixed in some way since the number of the small x data is not sufficient to determine unambiguously their values from the fits.

Alternative logarithmic parametrizations

$$F_2(x, Q^2) \sim \sum_{i=0}^2 b_i(Q^2) \ln^i \left(\frac{1}{x} \right) \quad (3)$$

exist and are claimed [5] to be equally efficient.

Notice also that in expressions (2),(3), contrary to (1), the Q^2 dependence factorizes in each individual term (Reggeon) - a typical feature of the Regge pole theory - (for more details see [4]).

It should be noted that each term in the simple parametrizations of the type (2) and (3) may be associated with the Q^2 independent Pomeron trajectories with relevant factorized Q^2 dependent residues. Formally, it is not compatible with the GLAP evolution equation, by which the variation (evolution) with Q^2 modifies also the x dependence of the SF (although in a limited range, approximate "selfconsistent" solutions, stable with respect to a logarithmic behaviour are known [6] to exist). In any case, since we are fitting the SF to fixed values (bins) of Q^2 , our parametrization does not depend directly on the Q^2 evolution. Moreover, since the onset and range of the perturbative GLAP evolution is not known *a priori*, our "data" may be used as a test for it.

In this paper we present the results of a comparative analysis of these two types of parametrizations : power-like and logarithmic. To avoid theoretical bias, we do not constrain the Q^2 dependence by any particular model, instead we take the experimental value in a parametric way.

The range of variables and the set of experimental points are, of course, the same in both cases. As a by-product, the parameters obtained in this way may be used as "experimental data" in future calculations of the GLAP or BFKL evolution. The present study is an extension of a preliminary analysis [6].

It is generally believed that, at small x , the singlet SF increases monotonically, indefinitely, accelerating towards larger Q^2 (the Pomeron becomes more "perturbative"). This phenomenon is usually quantified by means of the derivative $\partial \ln F_2 / \partial \ln(1/x)$, which in the simple case of $F_2 \sim x^{-\lambda}$ (λ being x independent), is identical with the effective power λ (otherwise it is not). We found evidence against this monotonic trend: moreover, we show that, at the highest Q^2 , the rise of F_2 starts slowing down.

2 Analysis of the structure function

2.1 Small x (< 0.05)

The following forms of the small x singlet component $(S, 0)$ of the SF are compared for $x < x_c$ and for each experimental Q_i^2 bin :

A. Power-like

$$F_2^{S,0}(x, Q_i^2) = a(Q_i^2) \left(\frac{1}{x}\right)^{\lambda(Q_i^2)}, \quad (4)$$

and

$$F_2^{S,0}(x, Q_i^2) = a_0(Q_i^2) \left(\frac{1}{x}\right)^{\epsilon_0} + a_1(Q_i^2) \left(\frac{1}{x}\right)^{\epsilon_1}, \quad (5)$$

where the exponents ϵ_0, ϵ_1 are fixed in accordance with [2, 3].

B. Logarithmic

$$F_2^{S,0}(x, Q_i^2) = b_0(Q_i^2) + b_1(Q_i^2) \ell n\left(\frac{1}{x}\right), \quad (6)$$

$$F_2^{S,0}(x, Q_i^2) = b_0(Q_i^2) + b_2(Q_i^2) \ell n^2\left(\frac{1}{x}\right) \quad (7)$$

and the combination of the two

$$F_2^{S,0}(x, Q_i^2) = b_0(Q_i^2) + b_1(Q_i^2) \ell n\left(\frac{1}{x}\right) + b_2(Q_i^2) \ell n^2\left(\frac{1}{x}\right). \quad (8)$$

In these equations, $a(Q_i^2)$, $a_{0,1}(Q_i^2)$, $b_{0,1,2}(Q_i^2)$ and $\lambda(Q_i^2)$ are parameters fitted to each i^{th} Q^2 bin. More precisely, the free parameters are a and λ for (4), a_0 and a_1 for (5), b_0 and b_1 for (6), b_0 and b_2 for (7), b_0, b_1 and b_2 for (8).

The choice of the cut x_c is obviously crucial, but subjective. Balancing between x small enough, to minimize the large x effects, and x large enough to include as many data points as possible, we tentatively set, like in [6] $x_c = 0.05$ as a compromise solution.

Since one can be never sure of the choice of the boundary below which the non-singlet contribution $(nS, 0)$ may be neglected ⁵, we performed additional fits with the non singlet contribution included

$$F_2^{nS,0}(x, Q_i^2) = a_f(Q_i^2) x^{1-\alpha_f}, \quad (9)$$

with the intercept fixed as in [8], $\alpha_f = 0.415$ (*i.e.* only one free parameter, namely a_f is added).

2.2 Extension to all x (< 1.0)

To ensure that our fits do not depend on the choice of the cut x_c , we extend the previous analysis to larger values of x with relevant modifications of the SF. Namely, we multiply the singlet and subsequently the non singlet contributions by appropriate large x factors [8]. The resulting SF becomes

$$F_2(x, Q_i^2) = F_2^{S,0}(x, Q_i^2)(1-x)^{n(Q_i^2)+4} + F_2^{nS,0}(x, Q_i^2)(1-x)^{n(Q_i^2)}, \quad (10)$$

⁵The ratio of the non-singlet contribution to the singlet one was calculated in [7] and was shown to drop below 10% around $x = 10^{-3}$, tending to decrease with increasing virtualities Q^2 .

where $F_2^{S,0}$ runs over all the cases considered in the previous section, and the exponent $n(Q_i^2)$ is either that of [8]

$$n(Q_i^2) = \frac{3}{2} \left(1 + \frac{Q_i^2}{Q_i^2 + c} \right), \text{ with } c = 3.5489 \text{ GeV}^2, \quad (11)$$

or is fitted to the data for each Q_i^2 value (see below).

3 Discussion of the results

3.1 Structure function

We made two kinds of fits, one restricted to small x only, ($x < x_c = 0.05$), the other one including large x as well. In the first case ($x < x_c$) the experimental data are from [9]. Altogether 43 representative Q^2 values were selected to cover the interval $[0.2, 1200] \text{ GeV}^2$ and $x \in [2.10^{-6}, x_c]$. Including more (or all available) data points had little effect on the resulting trend of the results. The relevant values of χ^2 , with and without the non-singlet term, are given in Table 1. Notice, that we use the definition

$$< \chi^2/\text{dof} > = \frac{\sum_{i=1}^{N_{bin}} \left(\frac{\chi_i^2}{n_{data_i} - m_{para}} \right)}{N_{bin}}, \quad (12)$$

where each Q_i^2 bin out of a total of N bins contains n data points and gives a resulting contribution to χ_i^2 in fitting eqs. (4)-(8), each containing m parameters.

Table 1. Results of the fits without or with non-singlet term (9) for small $x (< 0.05)$. The total number of experimental points is 508.

Version	Power	Power	Logarithm	Logarithm	Logarithm
Eq.	(4)	(5)	(6)	(7)	(8)
Nb. of parameters	2	2	2	2	3
χ^2	282	262	463	303	231
$< \chi^2/\text{dof} >$	0.68	0.62	1.04	0.70	0.61
Eqs.	(4,9)	(5,9)	(6,9)	(7,9)	-
Nb. of parameters	3	3	3	3	-
χ^2	247	225	253	234	-
χ^2/dof	0.67	0.60	0.67	0.62	-

Notice that in performing the small x fit we profited from a large set of available data, while in the large x extension a representative set of 30 Q^2 bins ($Q^2 \in [1.5, 2000] \text{ GeV}^2$) was used. The data are from [9, 10]. The relevant χ^2 values are shown in Table 2. Two options are presented : the first one relies entirely on the extension by [8], in the second one the exponent $n(Q^2)$ (see (10)) is fitted for each Q^2 bin.

Table 2. Results of the fits for all x (< 1.0), when the parameters of the large x extension n is chosen as in [8] or fitted. The total number of experimental points is 545.

Version	Power	Power	Logarithm	Logarithm	Logarithm
Eqs.	(4,9,10,11)	(5,9,10,11)	(6,9,10,11)	(7,9,10,11)	(8,9,10,11)
Nb. of parameters	3	3	3	3	4
χ^2	368	371	894	399	319
$< \chi^2/\text{dof} >$	0.79	0.79	1.74	0.85	0.78
Eqs.	(4,9,10)	(5,9,10)	(6,9,10)	(7,9,10)	-
Nb. of parameters	4	4	4	4	-
χ^2	321	317	541	329	-
χ^2/dof	0.76	0.75	1.25	0.780	-

The Q^2 dependence of the parameters was shown in Figs. 1-3. We exposed the most representative results from the small x fit that may clarify asymptotic trends in the behaviour of the singlet SF (see [6] and the following discussion of the results). As already explained, the large x extension was intended merely to support the small x results.

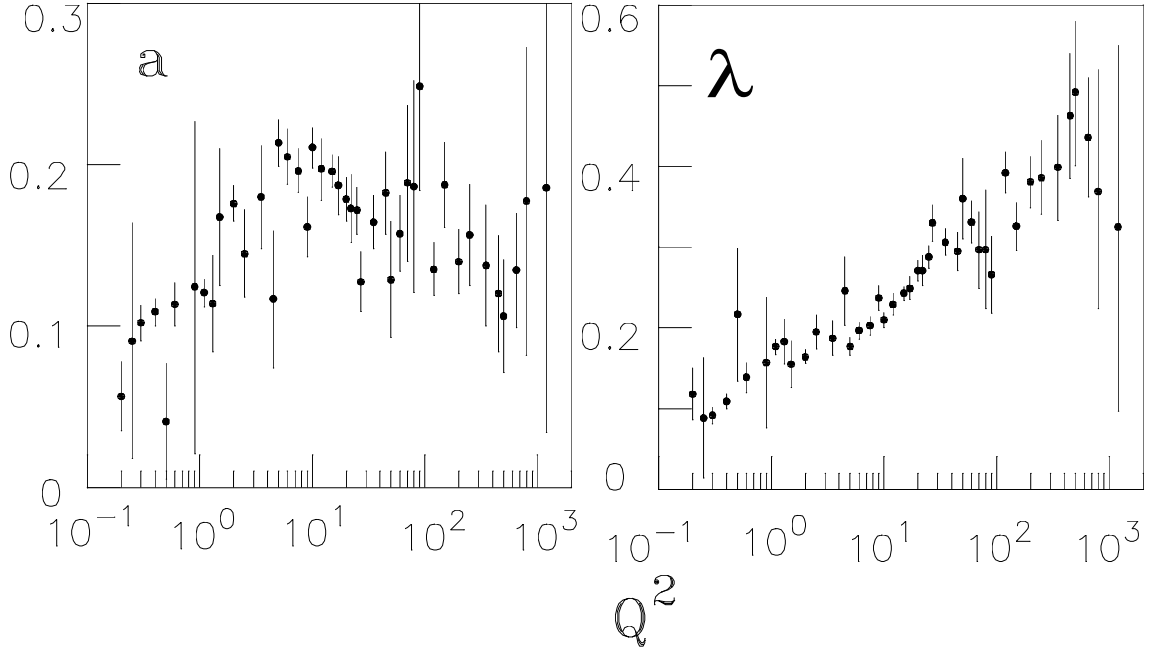


Fig. 1. Results of our analysis for $a(Q_i^2)$ and $\lambda(Q_i^2)$ entering in the parametrization (4) : $F_2^{S,0} = a(\frac{1}{x})^\lambda$ of the small x structure function ($x < x_c = 0.05$); they are fitted to the discrete values of Q^2 data from [9]; Q^2 is in GeV^2 , the error bars are produced from the minimization program "Minuit".

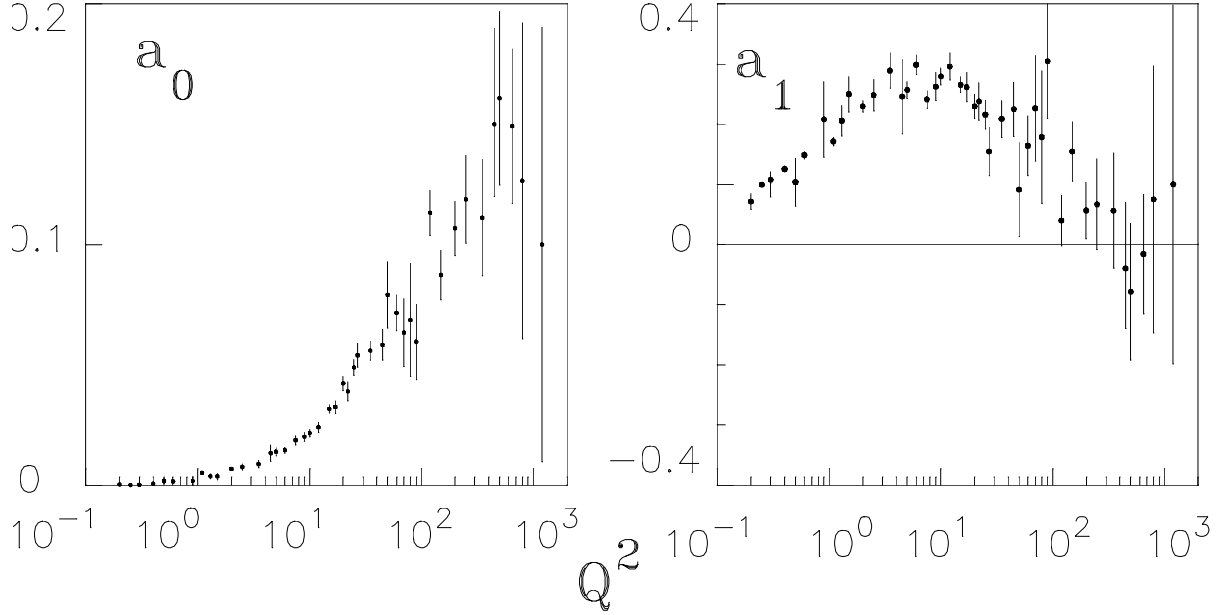


Fig. 2. Same as Fig .1 for $a_0(Q_i^2)$, $a_1(Q_i^2)$ and parametrization (5) :
 $F_2 = a_0(\frac{1}{x})^{0.418} + a_1(\frac{1}{x})^{0.0808}$.

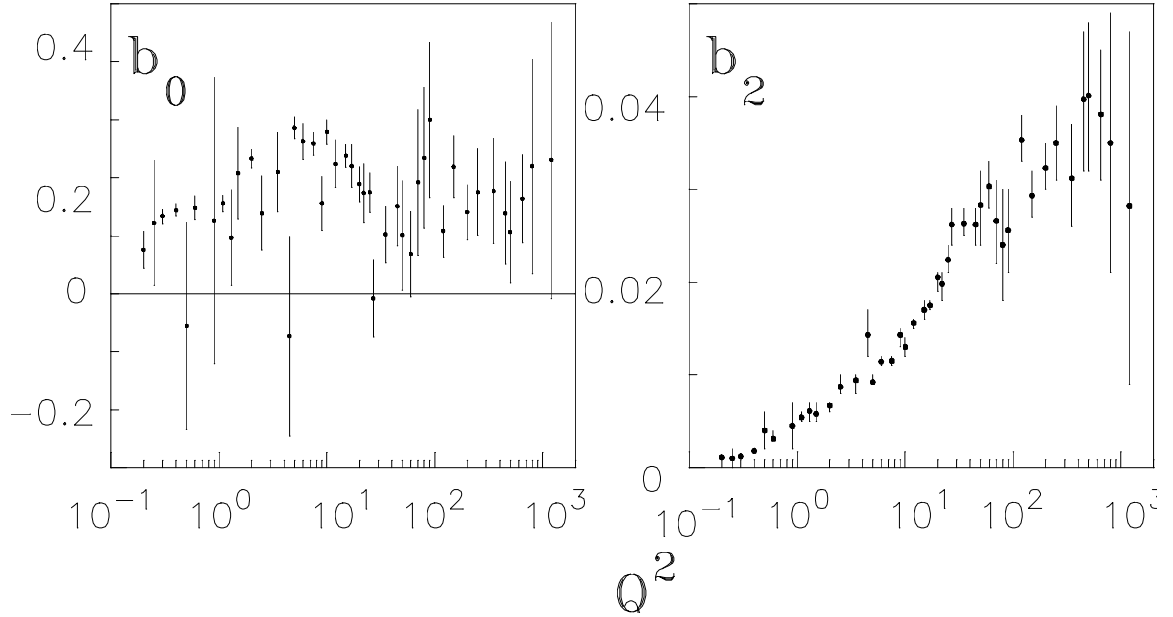


Fig. 3. Same as Fig .1 for $b_0(Q_i^2)$ and $b_2(Q_i^2)$ and parametrization (7) :
 $F_2^{S,0} = b_0 + b_2 \ln^2(\frac{1}{x})$.

The following comments are in order :

All the parametrizations (4)-(8), except (6), result in roughly equal quality fits. We may rule out the parametrization (6) giving the poorest (as expected) agreement with the data; so we do for the least economic (largest number of the free parameters), parametrization (8) (not shown in the figures).

The best results are achieved for the parametrization (8), giving the best value of the total χ^2 . Although fit (8) contains an extra free parameter with respect to the rest, the χ^2/dof value is, nevertheless, better than in other variants (4)-(7). Notice that (8) leads to alternating signs of the coefficients. Such an effect has been observed earlier in a fit to hadronic total sections [11]; it resembles the first few terms in an expansion of the supercritical Pomeron in an alternate series of logarithms.

3.2 x -slope

A clear indicator measuring the rate of increase of F_2 is its logarithmic derivative or the x -slope

$$B_x(x, Q_i^2) = \frac{\partial \ln F_2(x, Q_i^2)}{\partial \ln \frac{1}{x}} \quad (13)$$

identical with the effective power $\lambda(Q_i^2)$ in the case of a single power term, x independent as in (1) ⁶ (for an example of utilization of this derivative, see [12]).

The x -slope B_x is a function depending on two variables x and Q^2 , which in principle are independent, although correlated by a kinematical constraint: $y \leq 1$, which at HERA energy becomes Q^2 (in GeV^2) $< 9.10^4 x$. The derivative can be calculated either analytically, if the SF is parametrized explicitly, or numerically by calculating the finite difference within certain intervals $< x >$. If a given parametrization fits the data well, then the analytical differentiation has a chance to reflect the slope, although it will not be model-independent. By calculating the slopes of finite bins, we have a better chance to be model-independent, although the result may depend on the width of the chosen bins.

For the parametrization (4) $B_x = \lambda$ is already shown in Fig. 1. We show in Fig. 4 the results of our analysis corresponding to the other representative cases (5),(7), the coefficients of which are exhibited above.

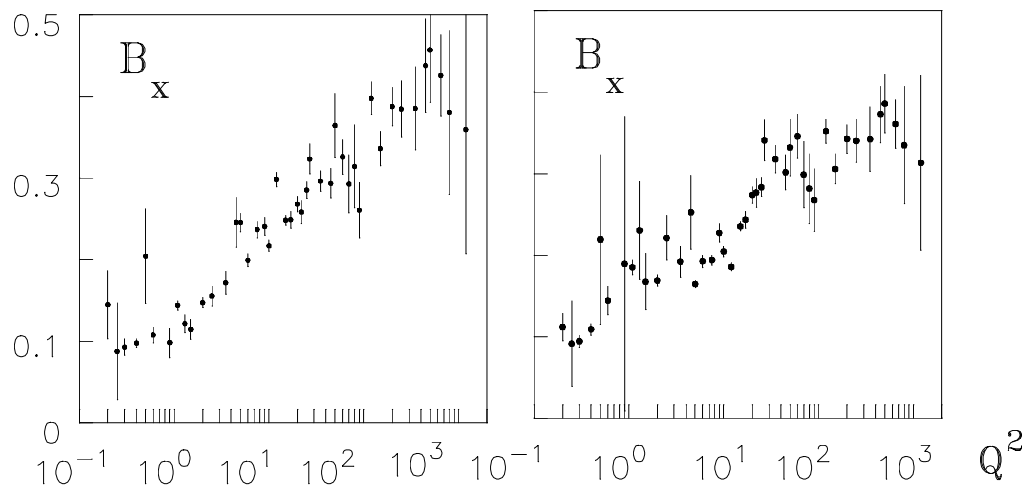


Fig. 4. x slope B_x versus Q^2 and parametrizations (5) : $F_2 = a_0(\frac{1}{x})^{0.418} + a_1(\frac{1}{x})^{0.0808}$ (left side) and (7) : $F_2^{S,0} = b_0 + b_2 \ln^2(\frac{1}{x})$ (right side).

⁶Note that by factorization, the intercept is Q^2 independent; that is why (1) is an "effective" Regge pole contribution rather than a genuine Pomeron [4].

Asymptotically, as $x \rightarrow 0$, the x -slope B_x calculated from eqs. (5)-(8) is Q^2 independent. However, for finite values of x , in the range of the present experiments, the Q^2 dependence, as it can be seen from Fig. 1,4 is still essential.

4 Conclusions

Our comparative analysis shows that several competing parametrizations for the small x structure function exist, providing equally good fits to the data. The Q^2 dependent intercept in (1) may be considered as an "effective" one, reflecting the contribution from two Pomerons in (2). Notice that logarithmic parametrization (3) conserving the unitarity bounds is equally efficient.

The fits show also some evidence that the rise of the singlet component of the structure functions with $1/x$ moderates as Q^2 increases, the turning point being around $Q^2 = 200 \text{ GeV}^2$, whenafter $F_2(x, Q^2)$ decelerates monotonically. Such a slow-down (deceleration) of the rate of increase was anticipated already in [13]. Later, it was confirmed and discussed in the framework of a model [14] interpolating (combining) between Regge behaviour and the high Q^2 asymptotics of the GLAP evolution equation. It was discussed also in [15] with a traditional Regge-type model with a Q^2 independent Pomeron intercept.

Apart from the turn-over in the x slope B_x , the "softening" of the singlet SF towards highest Q^2 may be visualized also from the behaviour of the fitted Q^2 dependent coefficients, namely a_1 in Fig. 2, and b_2 in Fig. 3.

Here, we only mention that the origin of the phenomenon - if confirmed - is either the increasing role of shadowing as Q^2 increases, restoring the Froissart bound, or the revelation of a contribution different from the "perturbative" Pomeron (whose role was believed to increase with increasing virtuality Q^2), or a combination of the two.

As discussed in [12], the concavity of the slope B_x with respect to Q^2 is another important quantity, indicative of the path of evolution (GLAP or BFKL).

Both (BFKL and GLAP) evolution equations are known to be the theoretical bases of the small x behaviour of the structure functions. While the perturbative solution is well known for the GLAP equation, its convergence for the BFKL equation is still debated. Approximate solution of both and the relevant path in the $x - Q^2$ plane may be revealed both from phenomenological models and from fits to the data.

Present data may already reveal [16] the actual path, but it should be remembered that the highest measured values of Q^2 do not reach the smallest x , so further measurement at possibly smallest x and highest Q^2 are eagerly awaited.

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